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Letter to the editor

A note on new exact solutions for the Kawahara equation using Exp-function method

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ABSTRACT

Exact solutions of the Kawahara equation by Assas [L.M.B. Assas, New Exact solutions for the Kawahara equation using Exp-function method, J. Comput. Appl. Math. 233 (2009) 97–102] are analyzed. It is shown that all solutions do not satisfy the Kawahara equation and consequently all nontrivial solutions by Assas are wrong.

Recently Assas in [1] looked for exact solutions of the Kawahara equation using the Exp-function method. This equation can be written as

$$u_t + \alpha u u_x + \beta u_{xxx} - \gamma u_{xxxxx} = 0, \tag{1}$$

where α, β and γ are constants.

The author [1] considered the Kawahara equation using the traveling wave $z = kx + wt$ and tried to find new exact solutions of the nonlinear ordinary differential equation

$$\gamma k^5 U_{zzzzz} - \beta k^3 U_{zzz} - \alpha k U U_z - w U_z = 0. \tag{2}$$

He believed that he found a few exact solutions of Eq. (2) but this is not the case. It is obvious that the author [1] cannot obtain any new nontrivial solutions of Eq. (2) using his approach. The matter is the general solution of Eq. (2) has the pole of the fourth order. This fact has an important bearing on the choice of expressions in the Exp-function method. However the author does not take this result into account. To look for exact solutions of the Kawahara equation the author [1] must take at least five terms in the denominator and numerator of the ansatz in the Exp-function method. We can see that all constructions by the author (formulae (15), (21) and (27) of work [1]) have poles of the second and third order.

Consequently we can expect that Assas cannot find any solutions of the Kawahara equation. Just to be on the safe side we have checked all solutions by Assas [1] and obtained that all solutions [1] do not satisfy the Kawahara equation except solution (17) that is the trivial solution $u = \frac{a_{-1}}{b_1}$. Naturally solutions (18), (19) and (20) are trivial solutions as well because they were obtained from solution (17).

We were surprised when we read that the author [1] has compared the accuracy of his exact solutions with known exact solutions and obtained that his results are "very precise and that there is an inverse relationship between distance and time". In conclusion, the author claims that "some new generalized solitary solutions with parameters are obtained".

In fact solitary and periodic solutions of the Kawahara equation were obtained many years ago (see for example [2–4]). Let us demonstrate that we can find the solitary wave solutions of the Kawahara equation using the tanh-method. Assuming $U(z)$ in the form

$$U(z) = A_0 + A_1 \tanh(z) + A_2 \tanh^2(z) + A_3 \tanh^3(z) + A_4 \tanh^4(z), \quad z = kx + wt - z_0 \tag{3}$$

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and substituting (3) into Eq. (2) we have

$$A_4=\frac{1680\gamma k^4}{\alpha},\qquad A_3=0,\qquad A_2=-\frac{280k^2(104\gamma k^2+\beta)}{13\alpha},\qquad A_1=0,$$
$$A_0=\frac{26\,4992\gamma^2k^5+7280\beta\gamma k^3-31\beta^2k-507\gamma w}{507\gamma\alpha k}. \tag{4}$$

As a result we obtain the solitary wave solutions in the form

$$u(x,t)=\frac{26\,4992\gamma^2k^5+7280\beta\gamma k^3-31\beta^2k-507\gamma w}{507\gamma\alpha k}-\frac{280k^2(104\gamma k^2+\beta)}{13\alpha}\tanh^2(z)+\frac{1680\gamma k^4}{\alpha}\tanh^4(z),$$
$$z=kx+wt-z_0 \tag{5}$$

with the following values of k:

$$k_{1,2}=\pm\frac{\sqrt{13\gamma\beta}}{26\gamma},\qquad k_{3,4}=\pm\frac{\sqrt{65\gamma\beta\left(-31+3\mathrm{i}\sqrt{31}\right)}}{260\gamma},$$
$$k_{5,6}=\pm\frac{\sqrt{-65\gamma\beta\left(31+3\mathrm{i}\sqrt{31}\right)}}{260\gamma}. \tag{6}$$

Other solutions in the form of the solitary waves of the Kawahara equation are not known. We are also sure that we cannot find other solitary wave solutions of the Kawahara equation because there are meromorphic solutions only in these cases. Unfortunately the author [1] made a few mistakes that were discussed in recent papers [5–9].

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